STUDY, REALIZATION AND APPLICATION OF CONSTANT PHASE ELEMENTS

PhD Brainstorming Day – 29th Oct. 2019

Dipartimento di Ingegneria Elettrica Elettronica e Informatica
Dottorato di Ricerca in Ingegneria dei Sistemi, Energetica, Informatica e delle Telecomunicazioni - XXXIV Ciclo
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Fractional-order Calculus

In the 1695 in a letter written by de L'Hopital and sent to Leibniz, the sender asked to Leibniz what would happen if the n-derivative of a function was of non-integer order, like $n = {}^{1}/_{2}$, instead of an integer value. The Leibniz's answer was the following: *"[...] This is an apparent paradox from which, one day, useful consequences will be drawn [...]"*. It might say that the study of fractional-order systems, FOSs, started in that time.
In general, n-derivative or n-integral calculus is commonly based on integer numbers, i.e., $n \in \mathbb{N}_0$, but what do happen if n is not an integer, i.e., $n \in \mathbb{R}$?
The integer-order calculus is a particular case of the fractional one. FOSs have a lot of applications: model and control of natural physical systems, human brain and its nervous system, diffusive phenomena in chemical reactions and so on. The main advantage of the fractional order is the **extra degree of freedom** that can be used to control the system's performance.

The general fractional-order operator can be defined according to the value of α [1]:
$$ {}_a\mathcal{D}_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & : \alpha > 0 \\ 1 & : \alpha = 1 \\ \int_a^t (d\tau)^{-\alpha} & : \alpha < 0 \end{cases} $$

Three different definitions for a fractional-order derivative (or integral) can be evaluated for a continuous-time function $f(t)$, [1]:

- **Caputo (C):** $${}^{C}_{a}\mathcal{D}_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau$$
- **Riemann-Liouville (RL):** $${}^{RL}_{a}\mathcal{D}_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau$$
- **Gründwald-Letnikov (GL):** $${}^{GL}_{a}\mathcal{D}_t^\alpha f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t-jh)$$

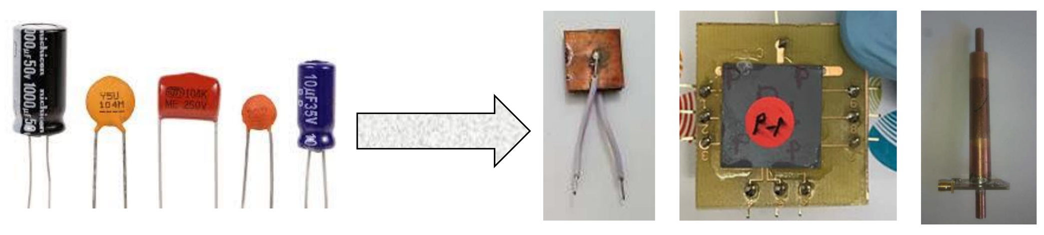
Constant-Phase Element

The implementation of a fractional-order transfer function is an hard demanding task. Two possible strategies arise:

1. **Approximation**: many methods to approximate a fractional-order transfer function into an integer-order one are defined (Oustaloup, [2], Charef, [3],…). → **complex structure and use of a lot of passive components**.
2. **Realization of fractional-order element (FOE)**: new technologies can be used to build components which have an intrinsic fractional-order behavior.

Following this last strategy, a new kind of electronics is required. FOEs can be realized considering different techonologies, [4]:

- Liquid electrode based;
- Geometry-related fractal;
- RC ladders;
- CMOS-based;
- Polymeric composite.



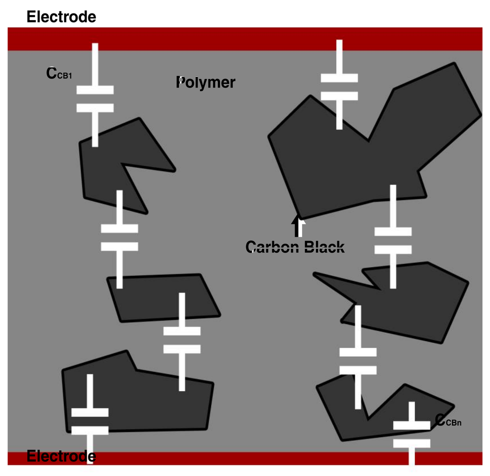
$$Z(s) = \frac{1}{sC} \qquad Z_{CPE}(s) = \frac{1}{s^\alpha C}$$

If the FOE can be represented by the transfer function Z_{CPE} and $\alpha \in \mathbb{R}$ it can be refered to as **Constant Phase Element**. It has an arbitrary constant phase in all its frequency domain.
In reality, the phase of CPE is not constant in all their frequency range due to their physical realization: a great challenge is to realize new technologies to build CPEs in a more wider frequency domain.

In our laboratory, two different kind of CPEs have been built: the first one exploits carbo-black nanoparticles, the second one the Bacterial Cellulose, a kind of cellulose produced by bacteria.

Carbon black-based FOE

This device has been built exploiting a cupper cylindrical capacitor, whose dieletrics has been realized considering a polymeric matrix in Sylgard184. Different percentages of carbon-black (CB) were diffused inside in order to change the diffusion process. Besides, different curing times at higher and different temperatures were chosen taking into account the manufacturer recommendation, see [5] for details.
Some of the realized deviced are depicted as follows.

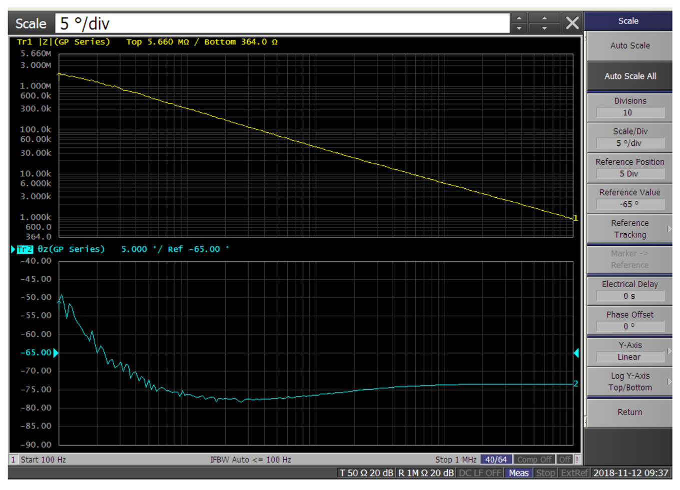


Name	CB %	Curing Temp	Curing Time
C – 125	8 %	125 °C	38 min
C – 140A	8 %	140 °C	32 min
C – 140B	**8 %**	**140 °C**	**32 min**
C – 150A	8 %	150 °C	28 min
C – 150B	8 %	150 °C	28 min
C – 160	8 %	160 °C	24 min

Considering the CB-140B device, its impedance has been measured using the Keysight Network Analizyer E5061B in the frequency range $[100;\ 1000000]\ Hz$. As it is well-evident, it has a **constant phase** almost equal to -75° in the range $[10;\ 1000]\ kHz$: here this device can be considered as a **CPE**.
The Genetic Algorithms (GAs), [6], have been used to identify its parameters, fitting the real data with the equivalent model Z_{CPE}. Its parameters are:

$$\alpha \cong 0{,}81;\ C \cong 2{,}7\ n\,{}^{F}/_{s^{1-\alpha}}$$

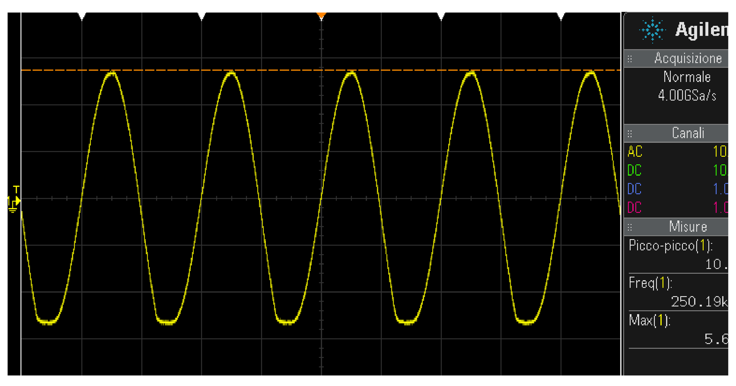
Using these values has been possible to test the aforementioned FOE in different electronics circuit to validate its behaviour.

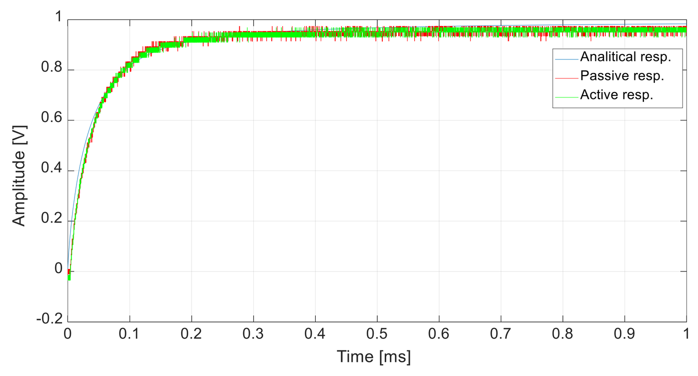


- **Fractional-order RC filter**

The output of the FO-RC filter can be evaluated as follows:

$$\frac{d^\alpha}{dt^\alpha} y(t) = -\frac{1}{RC} y(t) + \frac{1}{RC} u(t)$$





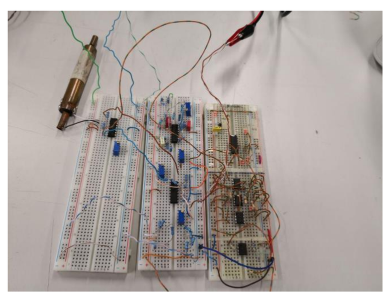
- **Fractional-order Wien Oscillator**

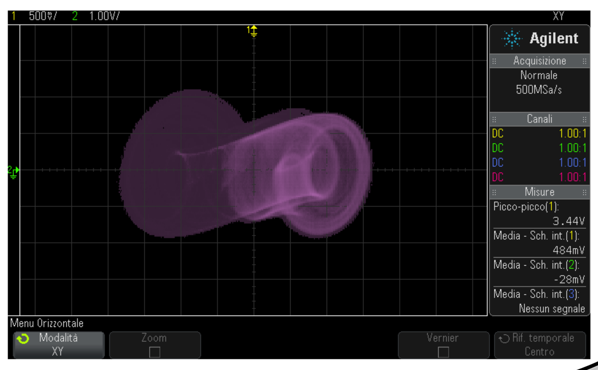
In [] has been demonstrated that a fractional-order oscillator, that still fulfils the Barkhausen condition, can be realized. In this application, a fractional-order oscillator with one CPE and one common capacitor has been realized.

- **Fractional-order Duffing system**

Duffing system, [7], is a second order nonlinear system able to show any kind of dynamical behaviour, i.e. from periodic oscillations to chaos, if opportunely excited.

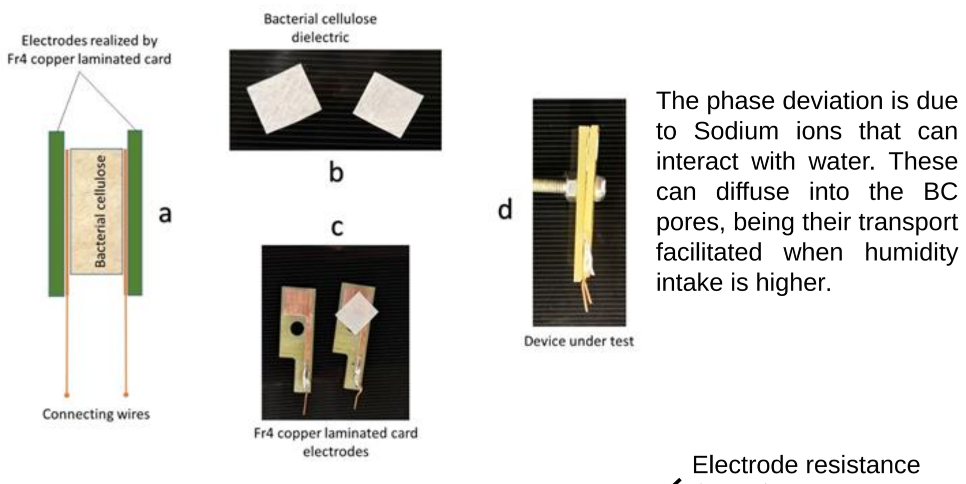
$$\begin{cases} \frac{d^\alpha}{dt^\alpha} x = y \\ \frac{d}{dt} y = x - x^3 - \delta y + \gamma \cos(\omega t) \end{cases}$$



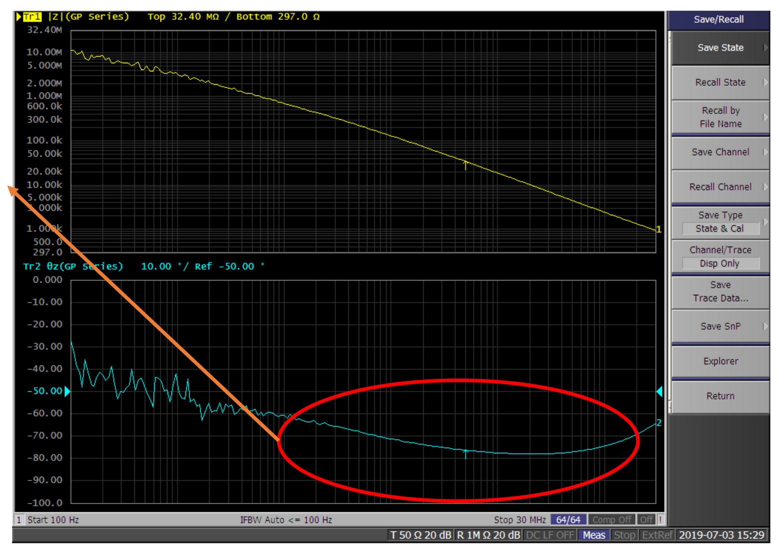


Bacterial Cellulose-based CPE

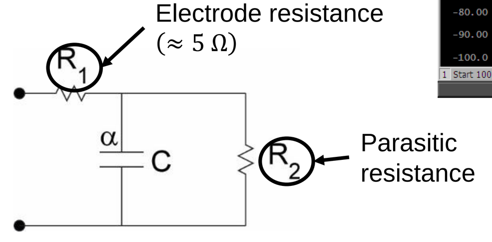
Bacterial cellulose (BC) is an organic compound with the formula $(C_6H_{10}O_5)_n$ and it is generated by some bacteria species, like *Acetobacter*, *Agrobacterium* or *Sarcina*.
Considering that BC has the same formula of the plant cellulose produced from the wood pulp, the former shows very different macromolecular structure and unique proprieties, such as high tensile strength, high cristallinity, high purity, water holding capability, [8]. Besides, BC can be obtained with a green and low-energy production process, which does not produce pollutants nor carbon composites. BC is used as the bulk in a capacitor-like structure. In this way a **green-electronics device** can be easily obtained.
The device has been realized as follows:



The phase deviation is due to Sodium ions that can interact with water. These can diffuse into the BC pores, being their transport facilitated when humidity intake is higher.

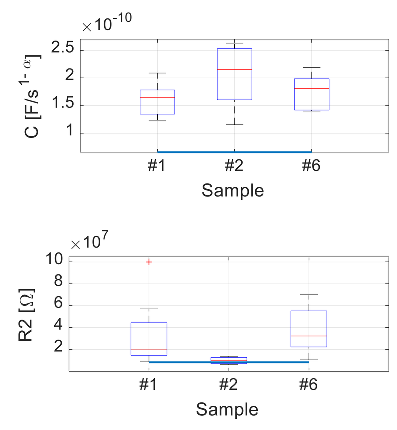


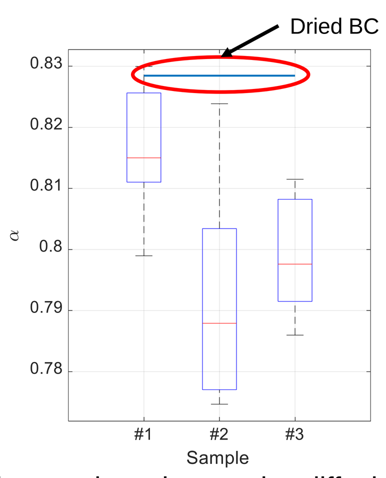
- **Electrical characterization**

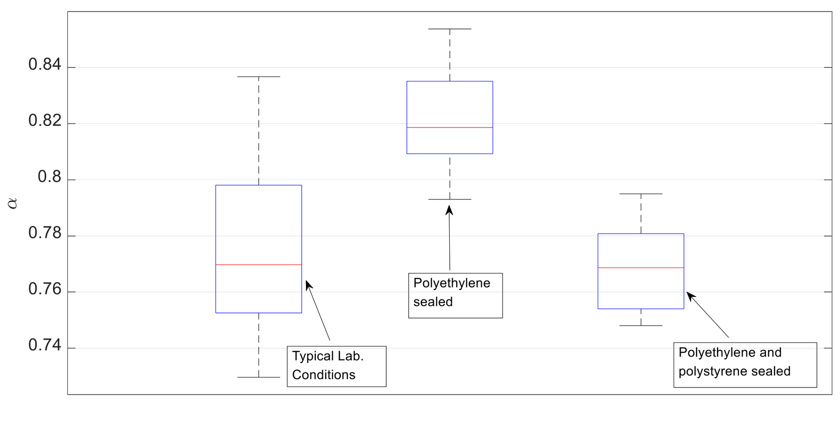


$$Z(s) = R_1 + \frac{R_2}{1 + s^\alpha R_2 C}$$

The water is responsible for BC's "fractionality".

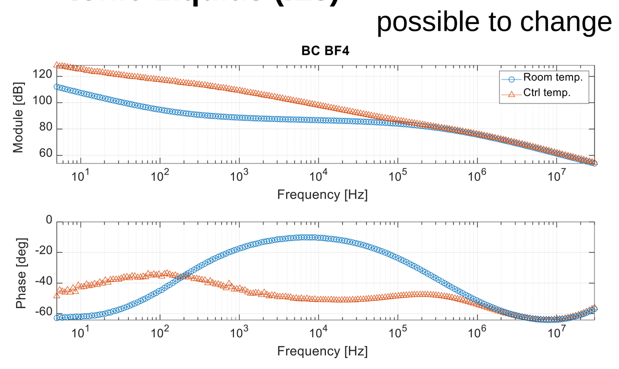


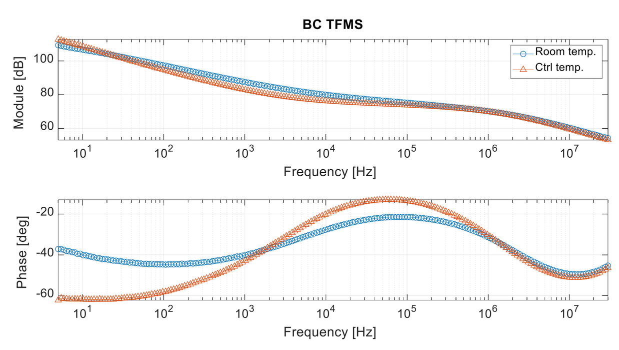


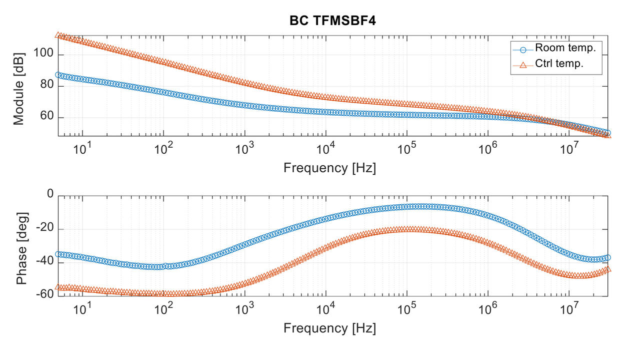


- **Ionic Liquids (ILs)**

ILs can be used to change the diffusion charges inside the BC. Considering their different nature it is possible to change the behavior of the BC-based device.







Future Work

CB140B:

- Study and describe in details the diffusion structure of the particles;
- Change the polymeric matrix inside the dielectrics in order to study the dependence of the material with respect to the α-order (on going);
- Change the diffused particles inside the dielectrics, using graphene or other materials (on going);
- Define a new productive process in which the device behaves as CPE at lower frequencies.

Bacterial Cellulose

- Study and define the dependence of the material with respect to temperature and humidity;
- Verify if it is possible to mix different ILs in different percentage in order to find, if it exists, a mathematical law that describes how changes the fractional-order of the device;
- Control the behavior of the BC-ILs at more adequate temperatures;
- Use these devices in real applications (e.g., electronics circuits and so on);
- Deposite flexible conducting polymers for realizing all-polymeric BC-based devices.
- Perform a multiphysics simulation to better understand how the charges, ILs in particulare, behave inside the BC's porous structure.

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